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Leigh McAlister
Sloan School of Management
Working Paper #1458-83
Massachusetts Institute of Technology
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ABSTRACT

Manufacturers are not necessarily in the "prisoner's dilemma" with their promotional spending. A small share brand may be made better off by promoting even if its behavior triggers a promotional response from larger share conpetitors. Given the seeming inevitability of promotional activity, we consider the management of promotional expenditures across a product line. A manufacturer with two brands, B_1 and B_2 (where B_1's market share is larger than B_2's), should make its decision whether to promote B_1, B_2 or both on the basis of the ratio of the promoted to unpromoted profit per unit. If that ratio is very small (a large percentage of the profit per unit is sacrificed in order to promote the product), only B_2 (the smaller share brand) should be promoted. For higher ratios of promoted to unpromoted profit, the manufactuer should promote only B_1 (the larger share brand). When the ratio of promoted to unpromoted profit is very high (almost no profit per unit is sacrificed in order to promote the product) both B_1 and B_2 should be promoted. Since continuous promotion ceases to be effective the manufacturer is also faced with the problem of sequencing its brand's promotions. For many product classes, the level of competitive promotional activity varies across time. A manufacturer whose brands are large relative to competitive promotional activity should promote its stronger brand in the season of heaviest competitive promotional activity. A manufacturer whose brands are small relative to competitive promotional activity should concede the heavy promotional season and promote its stronger brand in the season with lower competitive promotional activity.

A THEORY OF CONSUMER PROMOTIONS: MANAGERIAL IMPLICATIONS

Promotion expenditures have grown dramatically in recent years, reaching an estimated $60 billion in 1982.[1] Industry reactions to this trend are mixed. Hertz chair, Olson, finds such expenditures counter profitable.[2] 7-Up's chair, Winter, finds them essential and productive.[3] Bystanders attribute the confused spending in part, at least, to the management of promotion dollars. An Ad Age article titled "Room at the Top in Promotions," suggest that promotion management has remained clerical despite the fact that promotion expenditures have grown from 5% to 60% of a typical marketing budget. In "Sales Promotion - Fast Growth, Faulty Management," Strang (1976) suggests that promotion expenditures have been used by brand managers to optimize their career paths rather than their firms' profits.

In this paper we will explore the popular myth which suggests that promotional spending is inherently irrational since it places manufacturers in a situation analogous to the "prisoner's dilemma." We then develop rules of thumb that a profit maximizing manufacturer can use to allocate promotional dollars across products in a line and to sequence promotions through time.

We begin by reviewing existing literature that concerns itself with promotion management.

Literature Review

Blattberg, Eppen and Lieberman (1981) explore the hypothesis that retailers offer price deals in order to pass along inventory holding costs to consumers. The motivation for dealing, they contend, rests on the willingness of some consumers to stockpile the promoted product. The authors reject the hypothesis that retailers deal to attract customers from other stores. Their argument holds that such motivation would place retailers in "the prisoner's dilemma."[4] Their theoretical model, based on inventory theory, generates testable hypotheses with managerial implications. One such point dictates that larger share brands should be dealt more often.

Fraser (1983) ignores stockpiling and proposes that brand switching is the motivation for promotions. She proposes that each brand have a randomized promotion strategy characterized by a probability distribution over promotion intensities. At the beginning of each period, each brand would reveal its promotion intensity for that period and the brand with the most intense promotion would capture all promotion-sensitive consumers. The range of feasible promotion intensities is assumed constant across all brands. The brand specific manufacturing costs, however, are assumed to decline with market share. Fraser (1983) solves this game for an equilibrium. Among her other results, she finds that brands with larger shares are able to offer larger deals more often (because of their lower production costs) and are therefore more successful with the promotion-sensitive consumers.

Narasimhan (1982) allows for both stockpiling and switching behavior on the part of consumers in his theoretical and empirical investigation of coupons as price discrimination devices. Embedded in that paper is a model for evaluating the profitability of a promotional offer. That model recognizes the trade off between losses to customers who would have purchased the product anyway and gains for customers induced by the promotion to make their purchase. The model, however, is far from operational.

Neslin and Shoemaker (1983) also incorporate both stockpiling and switching behavior in their model for evaluating coupon promotions. In the spirit of decision calculus, they incorporate managerial judgments when empirical data is not available. Their model evaluates the relative profitability of two or more promotional offers. It does not, however, address the question of whether or not promotion should occur at all. Klein (1981), on the other hand, proposes a model that speaks to exactly that question. Given that a promotion has been offered, consumers have responded and cost information is available, his model reports the magnitude of the profit or loss.

Reaching for generalizability, Guadagui and Little (1983) perform LOGIT analysis on UPC data for an entire product class. They are able to estimate a number of interesting parameters to build models for managing marketing expenditures.

McAlister (1983) took a different tack in her reach for generalizability. From a prior theory, she constructed analytical models of individuals' responses to promotions. She aggregated those individual models to develop a model of market response. That model was interrogated to discover a characterization of product classes in which promotions will

be profitable. In this paper we further interrogate her model to address several issues of managerial concern. We first address the question of whether incentives for promoting place manufacturers in the prisoner's dilemma. We then turn to the problems of allocating promotional dollars across a product line and sequencing promotions through time.

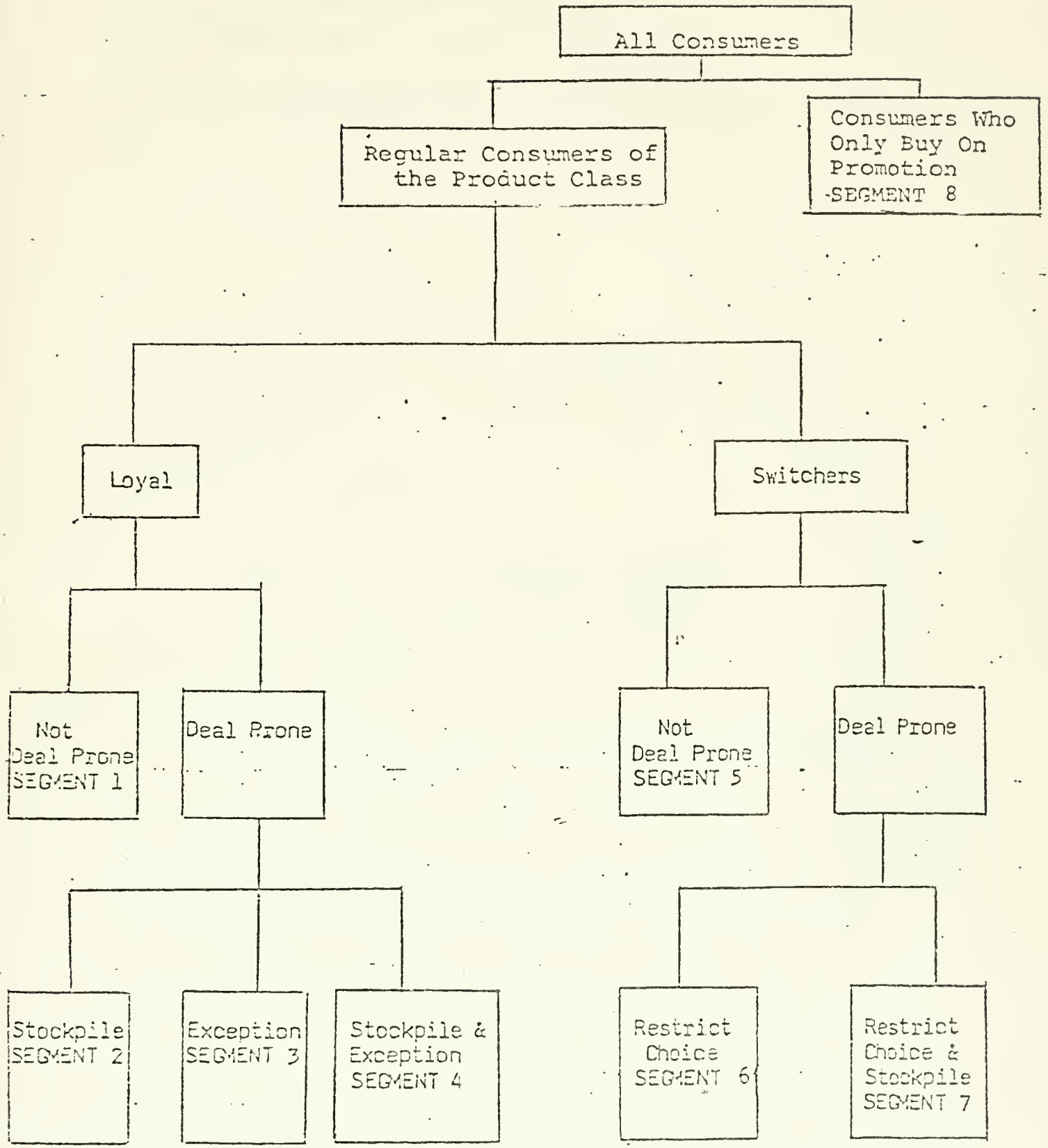
In the next section we briefly review the model as developed in McAlister (1983).

MODEL

McAlister (1983) begins with a series of assumptions regarding consumer response to promotions. She partitions consumers into eight segments as in Figure 1. Some consumers might only purchase in the product class if a promotion is offered (segment 8). Regular purchasers in the product class might be loyal (non-promoted purchases typically restricted to a single brand) or switchers (non-promoted purchases typically include two or more brands). Loyals or switchers might ignore promotions (segments 1 and 5 respectively) or they might respond. Loyal consumers might respond by stockpiling their favorite brand (segment 2), making an exception and purchasing a less preferred but promoted brand when their favorite is not being promoted (segment 3), or make an exception and stockpile that less preferred brand (segment 4). Switchers might respond to promotions by restricting their selection to just those brands which are being promoted (segement 6), or restricting their choice to promoted brands and stockpiling the brand selected (segement 7).

[FIGURE 1 ABOUT HERE]

FIGURE 1. SEGMENTATION SCHEME



Modeling the profit implications of each segments' purchasing behavior, McAlister (1983) arrives at a generic profit function:

$$G_s = \frac{\pi_s}{1+(n_s-1)(\alpha+\beta_s)} (\alpha\, k_s\, n_s\, M_2 + (1-\alpha-\beta_s)\, M_{1s}) \qquad (1)$$

$$G_T = \sum_{s=1}^{8} G_s S_s \qquad (2)$$

Where:

B_j = j^{th} brand in the market; j = 1, 2, ..., Z.

B_1 = brand whose profitability is being assessed.

G_s = gross profit to B_1 from a typical consumer in segment s.

S_s = number of consumers in segment s.

G_T = total gross profit for B_1.

P_1 = the proportion of loyal consumers loyal to B_1.

π_1 = the percent of time that switchers choose B_1.

π_s = P_1 , s = 1, 2, 3, 4

π_1 , s = 5, 6, 7, 8

n = the number of units that a stockpiling consumer will stockpile.

n_s = 1 , s = 1, 3, 5, 6, 8

n , s = 2, 4, 7

α = the proportion of time that B_1 is on promotion.

β = the proportion of time that some brand other than B_1 is on promotion.

β_s = 0 s = 1, 2, 5

β s = 3, 4, 6, 7, 8

P = coefficient of promotional leverage for loyals

$$= \frac{1}{\sum_{B_j \text{ on promotion}} P_j}$$

k = coefficient of promotional leverage for switchers

$$= \frac{1}{\sum_{B_j \text{ on promotion}} \pi_j}$$

$$k_s = \begin{cases} 1 & , s = 1, 2, 5 \\ p & , s = 3, 4 \\ k & , s = 6, 7, 8 \end{cases}$$

M_1 = regular profit per unit

M_2 = promoted profit per unit , $M_2 < M_1$

$$M_{1s} = \begin{cases} M_1, & s = 1, 2, \ldots, 7 \\ 0, & s = 8 \end{cases}$$

For a more detailed derivation and explanation see McAlister (1983).

By taking the derivative of equation (1) with respect to α, McAlister (1983) shows that promotion to segment 8 is always profitable, promotion to segments 1, 2 and 5 is never profitable, promotion to segments 3 and 6 is profitable if $k_s M_2 > M_1$ and promotion to segments 4 and 7 is profitable if $k_s (1+ \beta(n-1))M_2 > M_1$. A typical product class will be made up of a mix of consumers from many segments. The profitability of promoting to the entire mix of consumers is given by a weighted sum of the conditions for profitability in each segment.

In the later parts of this paper we focus on promotion policies for segments 3, 4, 6 and 7 whose profitability is ambiguous. The analytical structure of the profit equation for segment 3 parallels that of segment 6. (Replace p by k and P_1 by π_1, and they are identical). Similarly the analytical structure of the profit equation for segment 4 parallels that of segment 7. For expositional simplicity we will consier the profitability of promoting to exception segments (segments 3 and 6) as:

$$G_E = \pi_1 (\alpha k M_2 + (1-\alpha-\beta)M_1) \qquad (3)$$

and profitability of promoting to exception and stockpile segments (segments 4 and 7) as:

$$G_{ES} = \frac{\pi_1}{1+(n-1)(\alpha+\beta)} (\alpha k n M_2 + (1-\alpha-\beta)M_1) \qquad (4)$$

When applying the rules of thumb developed in this paper, a manufacturer should keep in mind that separate analyses should be done for loyals and switchers if $k \neq p$ or $\pi_1, \neq P_1$. Further, and importantly, the manufacturer must take into account the unambiguous drag on profitability imposed by segments 1, 2 and 5 and the unambiguous boost to profitability provided by segment 8. Equation (12) in McAlister (1983) provides an exact device for that accounting. In the later parts of this paper we assume that such analysis has been done and that promotion has been found to be profitable. We will concern ourselves with rules of thumb for choosing among various profitable options.

Before developing rules of thumb for managing promotions, however, we consider the question of whether manufacturers are in the prisoner's dilemma with their promotion expenditures. Is it the case that each manufacturer, individually, has an incentive to offer promotions, but if manufacturers as a group ceased promoting all together each would be made better off?

THE "MANUFACTURER'S DILEMMA"

The analysis in McAlister (1983) took as a given that competitors were engaging in promotional activity. It has been suggested (e.g., Shugan 1980) that competitors' sales preempted by a promoting brand are exactly offset by

that brand's loss in sales during competitive promotions. This line of reasoning suggests that each brand would achieve the same share of sales if all brands promoted efficiently or if no brands promoted at all. In the first case, promoted sales would yield only profit M_2 per unit. In the latter case, all sales would yield profit M_1.

Clearly, under this scenario all manufacturers are better off when no one promotes than when everyone promotes. However, there is an incentive for a manufacturer to begin promoting if no other manufacturers are promoting. The first manufacturer to promote would enjoy their maximum possible coefficients of promotional leverage. Furthermore, this manufacturer would not lose any sales to competitive promotions since competitors are, by assumption, not promoting.

The situation just described is analogous to the game theoretic construct called the "prisoner's dilemma." The situation as described by the model in equation (2) is slightly different yielding some interesting implications.

McAlister (1983) showed that the sign of $\partial G_T/\partial\alpha$ was not a function of α. An implicaton of that finding is that in those cases in which it is profitable to promote (i.e., $\partial G_T/\partial\alpha > 0$), one should promote as much as possible to promote (i.e., make α as large as possible). However, it was asserted that there is an exogenously determined upper bound on $\alpha = a$. All brands that are promoting should, therefore, be promoting with the same probability. This allows us to replace β in equation (2) by $(Z-1)$ a yielding:[5]

$$\hat{G}_T = \sum_{s=1}^{8} \frac{S_s \pi_s}{1 + (n_s - 1)(1 - Za)} (k_s n_s a \quad M_2 + (1 - Za) \quad M_{1s})$$

By examining $\frac{\partial \hat{G}_T}{\partial a}$ we can address the question of whether anyone should be promoting in this market.

$$\frac{\partial \hat{G}_T}{\partial a} = \sum_{s=1}^{8} \frac{S_s \pi_s}{[1+(n_s-1)(1-Za)]^2} \{k_s n_s - ZM_{1s}\}$$

Once again the critical derivative is a weighted sum of the conditions for profitability of promotions for each segment. However, the segment-specific conditions, $k_s n_s M_2 - ZM_{1s} > 0$, have changed in an important way. The conditions prescribed by taking the derivative of equation (2) took the probability of competitive promotions as exogenous and fixed. In this analysis that probability is tied to the probability that the brand under consideration is being promoted. That it is not profitable to promote to segments 1, 2 and 5 has not changed. It continues to be profitable to promote to Segment 8. The big difference is that promotion profitability in segments 3, 4, 6 and 7 hinge on

$$M_2/M_1 > Z/(k_s n_s)$$

To understand the stringency of this condition, consider first the case in which consumers don't stockpile ($n_s = 1$) and brand promotions don't overlap. Further, to simplify exposition, consider only switcher segments. This allows us to drop the s subscript and deal only with the k and π_1 notation. Implications for loyal segments follow directly by substituting p for k and P_1 for π_1.

We now have $k = 1/\pi_1$ and $Z/k = Z\pi_1$. If the proportions of consumers loyal to each brand are approximately equal and if switchers are approximately random in their selection of brands, then brand shares are

approximately equal and π's will approximately equal 1/Z. Hence $Z/k = Z\pi_1 \simeq Z(1/Z) = 1$. Since M_2, the promoted profit per unit, is by definition smaller than M_1, the regular profit per unit, the condition for profitability $M_2/M_1 > Z/k \simeq 1$ cannot hold. A situation such as this is analagous to the "prisoner's dilemma." Given that all brands will promote if any promote, it is in everyone's best interest to curb promotional activity.

However, if brand loyalties or switching proportions are very different in size, it could happen that it is only in <u>almost</u> everyone's best interest to curb promotions. A very small share brand (i.e., a brand for which $\pi_1 < M_2/(M_1Z)$ would find promotions attractive even if it meant that all competitors would promote. This follows from the fact that if $\pi_1 < M_2/(M_1Z)$, then $Z/k = Z\pi_1 < Z(M_2/(M_1Z)) = M_2/M_1$. Notice that this is <u>not</u> the type of incentive that drives the prisoner's dilemma. In the prisoner's dilemma each party could be made better off by unilaterally promoting when others abstained. However, if all parties promote then everyone is made worse off. In a world characterized by equation (2), a small share brand (i.e., $\pi_1 < M_2/(M_1Z)$), like all other brands, could be made better off by promoting when all others abstained. If all parties promote then everyone else is made worse off <u>but the small share brand is still better off</u>. When we consider the implications of stockpiling, the long run attractiveness of promotions extends to even larger brands. In that case it is in the best interests of brands for which $\pi_1 < (nM_2)/(M_1Z)$ to promote even if other brands begin promoting.

This situation arises because of the enormous sales gains that a small share brand can make when it promotes alone. Rather than capture its small share of segments 3, 4, 6 and 7, it can capture them completely. The effect is magnifed when stockpiling occurs.

For the case in which brand promotions do overlap, the incentives for "small" share brands just described are less likely to occur. Suppose the expected level of promotional competition at any point in time = K, $0 \leq K \leq 1$. That is, at any given point in time the sum of the preference intensities of brands being promoted is expected to equal K. Should an arbitrary brand, B_j, decide to promote, the expected level of promotional activity would rise to $K + \pi_j$ and the coefficient of promotional leverage, k, would drop from $\frac{1}{K}$ to $\frac{1}{K+\pi_j}$. The condition for profitable promotions would then become:

$$\frac{M_2}{M_1} > Z/(k\ n) = Z(\pi_j + K)/n$$

or

$$\pi_j < \{(n\ M_2) / (M_1 Z)\} - K$$

As the level of ambient promotional activity, K, grows larger, even the small share brand is unable to produce sufficient sales gains when promoting to offset sales losses when others promote.

In summary, if all brand shares in the market are approximately equal, it is in all brands best interests for all brands to forego promotions altogether. However, if one brand begins promoting and condition (2) holds, it becomes in every brand's best interest to engage in promotions. In this case, manufacturers find themselves in a situation like the "prisoner's dilemma."

The situation is fundamentally different if brand shares differ markedly in magnitude. A small share brand can be made better off by promoting even if it triggers a promotional response by larger competitors.

It is interesting to note the consistency of this paradigm with observed market behavior. It was the smaller car rental agencies who began the car rental promotions. It was small share Chrysler who began giving auto rebates. Dad's Rootbeer (a small player in the soft drink market) focuses their marketing resources on consumer promotions.[6]

For those situations in which promotions are profitable, the manager is faced with the problem of selecting which among the brands in her line should be promoted and how to sequence those promotions. In the next section, we sketch broad outlines for managing promotional spending.

RULES OF THUMB FOR MANAGING PROMOTIONS

This section begins with a statement of the assumptions underlying the "rules of thumb" to be presented. We then show that the selection of a brand to promote depends upon the ratio of the promoted to unpromoted profit (M_2/M_1). The optimal sequencing of promotions is shown to be a function of the sizes of the manufacturers brands shares relative to the level of ambient promotional activity.

Assumptions

Again we restrict outselves to the k, π notation for expositional simplicity as we consider the management by manufacturer 1 of promotions for two brands, B_1 and B_2, with π_1 and π_2 respectively; $\pi_1 > \pi_2$. Extension of these findings to product lines with more than two brands should not be difficult. The rules for assessing the relative attractiveness of promoting each of two brands can be successively applied to render a complete ranking of all brands.

The "expected level of promotional competition," K, was introduced in the section titled "The 'Manufacturer's Dilemma'." It represents manufacturer 1's expectation of the total of the purchase probabilities of all brands being promoted prior to manufacturer 1's promotion decision. Should manufacturer 1 decide to promote B_1, the level of promotional activity would rise to $K + \pi_1$. Promoting B_2 instead would raise promotional activity to $K + \pi_2$, etc. We will begin by assuming that K is constant across the planning period and address the question of whether B_1, B_2 or both should be promoted. We then consider the possibility of there being two promotional seasons in the planning period. In season I, promotional activity is expected to be heavier than in season II. K_I and K_{II} represent the levels of promotional activity in those seasons, $K_I > K_{II}$.

We assume that it would be profitable to promote B_1 or B_2 or both and that our problem is to decide which of the three options is most profitable. Furthermore we assume that the profitable segment (segment 8) and the unprofitable segments (segments 1, 2 and 5) are of negligible size.[7] We concern ourselves with those segments who respond to promotions by making exceptions to their choice rules (segments 3 and 6) and those who respond by making exceptions and stockpiling (4 and 7).

Some additional notation is needed to represent the profit resulting from various promotional strategies:

$P^i_{a,b}$ = profit from the "exception" segments (3 and 6) that results from promotion strategy (a,b) in season i,

$i =$ I for season I, II for season II, omitted for the analysis in which K is assumed constant

$a =$ 1 if B_1 is promoted; $\sim$1 if B_1 is not promoted

$$b = \begin{cases} 2 \text{ if } B_2 \text{ is promoted} \\ \sim 2 \text{ if } B_2 \text{ is not promoted} \end{cases}$$

$\hat{P}^i_{a,b}$ = profit from the "exception and stockpile" segments (4 and 7) that results from promotion strategy (a,b) in season i, with (a,b) and i as defined above.

Selecting a Brand to Promote

As will be demonstrated, the optimal promotion strategy to the "exception" segments (3 and 6) is a special case of the optimal promotion strategy to the "exception and stockpile" segments (4 and 7). For expositional purposes we begin with an analysis of the "exception" segments.

"Exception" Segments (3 and 6)

This analysis turns on an investigation of the impact of differing promotional strategies on k, the coefficient of promotional leverage.

Recall that $k = \frac{1}{\sum_{B_j \in \hat{B}} \pi_j}$. For this analysis we let $\sum_{\substack{B_j \in \hat{B} \\ B_j \neq B_1, B_2}} \pi_j = K$. Hence, the coefficient of promotional leverage resulting from the promotion of only B_1 is $k_1 = \frac{1}{\pi_1 + K}$. The coefficient of promotional leverage resulting from the promotion of only B_2 is $k_2 = \frac{1}{\pi_2 + K}$. And the coefficient of leverage resulting from the promotion of both B_1 and B_2 is $k_{1 \text{ and } 2} = \frac{1}{\pi_1 + \pi_2 + K}$. Given these definitions for the coefficient of promotional leverage corresponding to each of the possible promotional strategies, we define:

$$P_{1,\sim 2} = \pi_1\left(\frac{1}{\pi_1 + K}\,\alpha\, M_2 + (1-\alpha-\beta)M_1\right) + \pi_2(1-\beta)M_1$$

$$P_{\sim 1,2} = \pi_1(1-\beta)M_1 + \pi_2\left(\frac{1}{\pi_2+K}\,\alpha\, M_2 + (1-\alpha-\beta)M_1\right)$$

$$P_{1,2} = (\pi_1+\pi_2)\left(\frac{1}{\pi_1+\pi_2+K}\,\alpha\, M_2 + (1-\alpha-\beta)M_1\right)$$

If only one brand is to be promoted, the question of whether to promote B_1 or B_2 reduces to an investigation of the sign of $P_{1,\sim 2} - P_{\sim 1,2}$. If that quantity is positive, it is more profitable to promote B_1. If that quantity is negative, it is more profitable to promote B_2.

$$P_{1,\sim 2} - P_{\sim 1,2} = \alpha\left(\frac{\pi_1}{\pi_1+K} - \frac{\pi_2}{\pi_2+K}\right) M_2 - (\pi_1-\pi_2)M_1 \qquad (5a)$$

$$= \alpha(\pi_1-\pi_2)\left(\frac{K\, M_2}{(\pi_1+K)(\pi_2+K)}\right) - M_1) \qquad (5b)$$

Equation (5a) demonstrates that the choice of a brand to promote hinges on the trade off that occurs during the α of the time that the chosen brand will be promoted. The promotion strategy does not affect the pattern of profits for the other $1-\alpha$ of the time. The question is whether the increment of promotional profit generated by promoting B_1 rather than B_2, $\left(\frac{\pi_1}{\pi_1+K} - \frac{\pi_2}{\pi_2+K}\right) M_2$, can offset the related loss in regular sales profit, $(\pi_1-\pi_2)M_1$.

The increment in promotional sales from promoting B_1 rather than B_2 is $\frac{\pi_1}{\pi_1+K} - \frac{\pi_2}{\pi_2+K}$. Notice that $\frac{\pi_1}{\pi_1+K}$ is the height of B_1's promotional sales peak and $\frac{\pi_2}{\pi_2+K}$ is the height of B_2's promotional sales peak. The level of competitive promotional activity, K, affects these two peaks differentially. Both decrease monotonically as K increases. However, B_2's peak shrinks more rapidly than B_1's as the level of competitive promotion grows from 0 to $\sqrt{\pi_1\pi_2}$. B_1's peak shrinks more rapidly than B_2's as the level

of competitive promotion grows above $\sqrt{\pi_1\pi_2}$.[8] And, in fact, the difference $\frac{\pi_1}{\pi_1+K} - \frac{\pi_2}{\pi_2+K}$ is maximized when $K = \sqrt{\pi_1\pi_2}$.[9]

For (5a) to be positive, it must be true that:

$$(\frac{\pi_1}{\pi_1+K} - \frac{\pi_2}{\pi_2+K}) M_2 - (\pi_1 - \pi_2)M_1 > 0 \qquad (6a)$$

$$\rightarrow \frac{K M_2}{(\pi_1+K)(\pi_2+K)} - M_1 > 0 \qquad (6b)$$

Expression (6b), which is equivalent to (6a), can be rearranged to yield a boundary condition on the relative margins.

$$\frac{M_2}{M_1} > \pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K} \qquad (7)$$

When condition (7) is satisfied, B_1 should be promoted. When it is not satisfied, B_2 should be promoted.

Since $K = \sqrt{\pi_1\pi_2}$ maximizes the coefficient of M_2 in (6a), an upper bound for expression (6a) can be found by setting $K = \sqrt{\pi_1\pi_2}$. This translates into a lower bound for condition (7):

$$\frac{M_2}{M_1} > \pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K} \geq \pi_1 + \pi_2 + 2\sqrt{\pi_1\pi_2} \qquad (8)$$

Up to this point we have assumed that only one brand was to be promoted. We now consider the relative profitability of promoting both brands simultaneously.

The incremental profit which results from promoting both brands rather than just B_1 is

$$P_{1,2} - P_{1,\sim 2} = \alpha\, \pi_2 \{\frac{K M_2}{(\pi_1+\pi_2+K)(\pi_1+K)} - M_1\}$$

It will be more profitable to promote both B_1 and B_2 rather than just

B_1 if:

$$\frac{M_2}{M_1} > 2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K} \qquad (9)$$

As before, a lower bound to this boundary condition can be obtained. In this case the value of K that maximizes the coefficient of M_2 is $K = \sqrt{\pi_1(\pi_1+\pi_2)}$, hence

$$\frac{M_2}{M_1} > 2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K} \geq 2\pi_1 + \pi_2 + 2\sqrt{\pi_1(\pi_1+\pi_2)} \ .$$

The incremental profit which results from promoting both brands rather than just B_2 is:

$$P_{1,2} - P_{\sim 1,2} = \alpha\pi_1 \left\{\frac{K}{(\pi_1+\pi_2+K)(\pi_2+K)} M_2 - M_1\right\}$$

It will be more profitable to promote both B_1 and B_2 rather than just B_2 if:

$$\frac{M_2}{M_1} > \pi_1 + 2\pi_2 + K + \frac{\pi_2(\pi_1+\pi_2)}{K} \geq \pi_1 + 2\pi_2 + 2\sqrt{\pi_2(\pi_1+\pi_2)} \ .$$

The lower bound on the inequality is determined using $K = \sqrt{\pi_2(\pi_1+\pi_2)}$ which maximizes the coefficient of M_2 in this case.

We summarize these results in Table 1 which describes manufacturer 1's optimal promotional strategy as a function of its relative margins.

[TABLE 1 ABOUT HERE]

The suggested strategy is diagrammed for several different (π_1,π_2) pairs in Figure 2. Rather than select an arbitrary value for K in Figure 2, the break points are marked at the absolute lower limit

for each strategic change. These absolute lower limits correspond to the lower bounds derived for conditions (8) and (9). They should be interpreted as follows:

No matter what the value of K, for a given (π_1,π_2) pair it is better to promote only B_2 when M_2/M_1 falls in the region below the marked "Promote Only B_2"/"Promote Only B_1" breakpoint. For values of K much greater or much less than $\sqrt{\pi_1\pi_2}$, the true breakpoint will move to the right.

Similarly, no matter what the value of K, for a given (π_1,π_2) pair it is better to promote only B_1 if M_2/M_1 is above the true "Promote Only B_2"/"Promote Only B_1" breakpoint and below the marked "Promote Only B_1"/"Promote Both B_1 and B_2" breakpoint. For values of K much greater or much less than $\sqrt{\pi_1(\pi_1+\pi_2)}$, the true breakpoint will move to the right.

[FIGURE 2 ABOUT HERE]

Figure 2 suggests that manufacturers with very high share brands should promote the weaker brand unless that manufacturer's promoted to regular profit ratio, M_2/M_1, is very high. Conversely, manufacturers with very low share brands should promote both brands unless that manufacturer's profit ratio is very low. Manufacturers with brands with moderate shares should carefully evaluate the alternatives according to the conditions outlined in Table 1.

"Exception and Stockpile" Segments (4 and 7)

The analysis for these segments, too, turns on the impact of differing promotional strategies on k, the coefficient of promotional leverage. In this case, however, the profit implications for the trade off will extend beyond

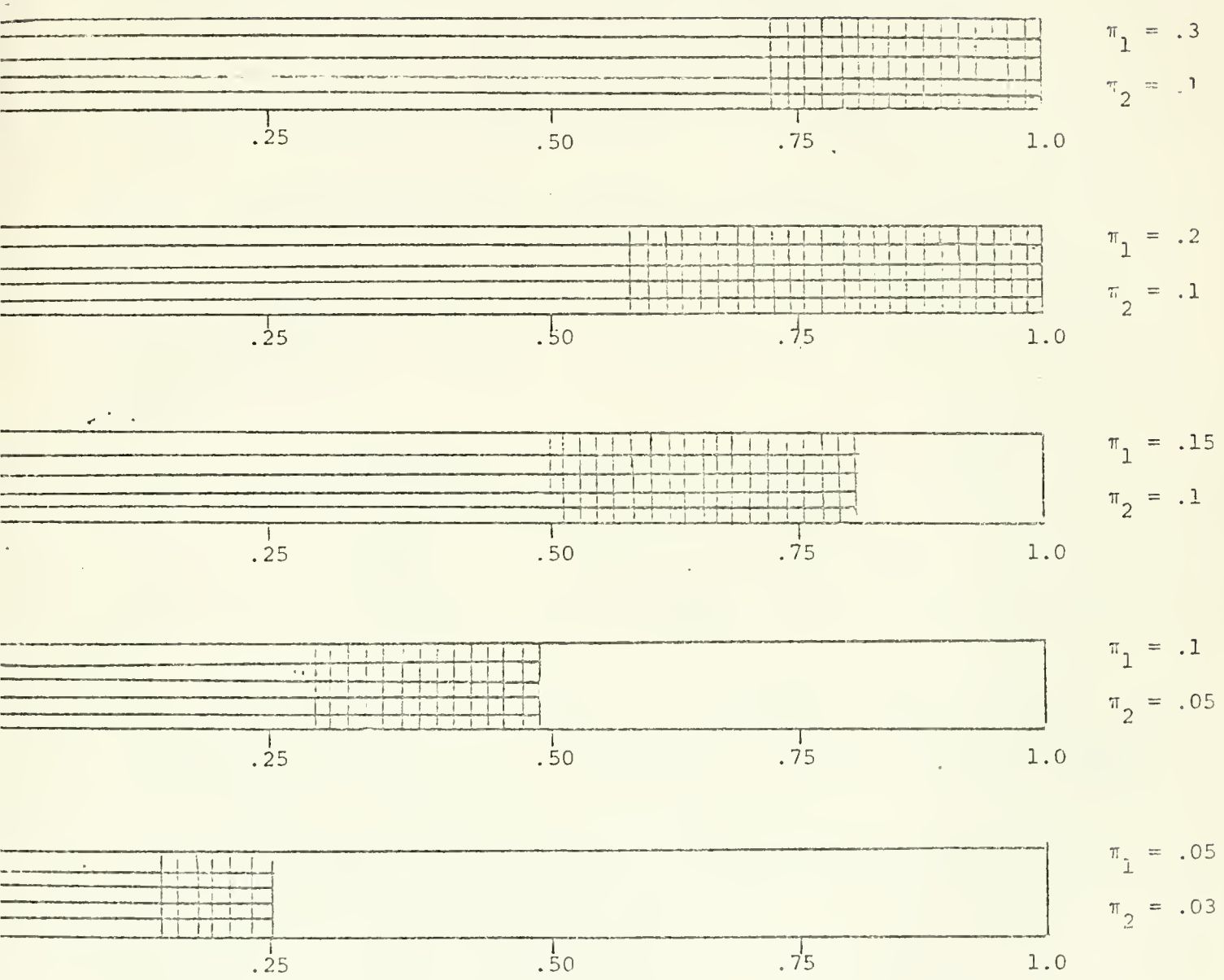
TABLE 1: OPTIMAL PROMOTIONAL STRATEGY TO SEGMENTS 3 AND 6

Conditions on M_2/M_1	Brand(s) to Promote
$\frac{M_2}{M_1} < \pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K}$	B_2
$\pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K} < \frac{M_2}{M_1} < 2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K}$	B_1
$\frac{M_2}{M_1} > 2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K}$	B_1 and B_2

FIGURE 2:

STRATEGY FOR PROMOTION TO SEGMENTS 3 AND 6 AS A FUNCTION OF $\frac{M_1}{M_2}$

FOR DIFFERENT VALUES OF π_1 AND π_2



that α of the time that the chosen brand (or brands) will be promoted. Since stockpiling occurs in these segments, the profit implications spill over into that $1-\alpha-\beta$ of the time when no brands are being promoted.

Similarly to the analysis of "exception" segments, we define:

$$\hat{P}_{1,\sim 2} = \frac{\pi_1}{1+(\alpha+\beta)(n-1)} \left(\frac{n\alpha M_2}{K+\pi_1} + (1-\alpha-\beta)M_1 \right) + \pi_2(1-\beta)M_1$$

$$\hat{P}_{\sim 1,2} = \pi_1(1-\beta)M_1 + \frac{\pi_2}{1+(\alpha+\beta)(n-1)} \left(\frac{n\alpha M_2}{K+\pi_2} + (1-\alpha-\beta)M_1 \right)$$

$$\hat{P}_{1,2} = \frac{(\pi_1+\pi_2)}{1+(\alpha+\beta)(n-1)} \left(\frac{n\alpha M_2}{K+\pi_1+\pi_2} + (1-\alpha-\beta)M_1 \right)$$

and

$$\hat{P}_{1,\sim 2} - \hat{P}_{\sim 1,2} = \frac{n\alpha(\pi_1-\pi_2)}{1+(\alpha+\beta)(n-1)} \left(\frac{K M_2}{(\pi_1+K)(\pi_1+K_2)} - M_1 \right) - (1-\alpha-\beta)\frac{\beta}{\alpha}\left(\frac{n-1}{n}\right) M_1 \qquad (10)$$

$$\hat{P}_{1,2} - \hat{P}_{\sim 1,2} = \frac{n\alpha\pi_2}{1+(\alpha+\beta)(n-1)} \left(\frac{K M_2}{(\pi_1+K)(\pi_1+\pi_2+K)} - M_1 \right) - (1-\alpha-\beta)\frac{\beta}{\alpha}\left(\frac{n-1}{n}\right)M_1 \qquad (11)$$

Notice the similarity of equations (5b) and (10). Both contain the expression $\frac{K M_2}{(\pi_1+K)(\pi_2+K)} - M_1$, whose sign drives the choice between promoting B_1 or B_2 for the "exception" segments. Equation (10) also carries an additional multiplier $\left(\frac{n}{1+(\alpha+\beta)(n-1)}\right)$ which arises because of stockpiling. This term merely magnifies the profit implications. It does not affect the sign of $\hat{P}_{1,\sim 2} - \hat{P}_{\sim 1,2}$ and therefore has no effect on strategy selection.

The final difference between (5b) and (10) is the term $(1-\alpha-\beta)\frac{\beta}{\alpha}\left(\frac{n-1}{n}\right)M_1$, which is subtracted in (10). This term captures the spillover effect of stockpiling behavior on manufacturer 1's profitability during that $1-\alpha-\beta$ of the time that no brands are being promoted. The

algebraic form has intuitive appeal. $(1-\alpha-\beta)$ indicates the extent of vulnerability. $\frac{\beta}{\alpha}$ represents the ratio of the level of competitive promotional activity to manufacturer 1's level of promotional activity. The smaller this number, the lower the losses due to stockpiling for manufacturer 1. $\frac{n-1}{n}$ grows as the number of units stockpiled grows, amplifying the competitive ratio effect.

A condition on relative profits for the "stockpile and exception" segments analogous to that in equation (8) for the "exception" segments suggests that B_1, rather than B_2, should be promoted if:

$$\frac{M_2}{M_1} > [1 + (1-\alpha-\beta)\frac{\beta}{\alpha}(\frac{n-1}{n})]\ (\pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K})$$

$$\geq [1 + (1-\alpha-\beta)\frac{\beta}{\alpha}(\frac{n-1}{n})]\ (\pi_1 + \pi_2 + 2\sqrt{\pi_1\pi_2}) \qquad (12)$$

The absolute lower bound for inequality (12) is derived by setting $K = \sqrt{\pi_1\pi_2}$ as described earlier.

Equations (6b) and (11) exhibit a pattern of similarity analogous to that just described for equations (5b) and (10). Analysis of equation (11) suggests that both B_1 and B_2, rather than just B_1, should be promoted if:

$$\frac{M_2}{M_1} > [1 + (1-\alpha-\beta)\frac{\beta}{\alpha}(\frac{n-1}{n})]\ (2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K})$$

$$\geq [1+(1-\alpha-\beta)\frac{\beta}{\alpha}(\frac{n-1}{n})]\ (2\pi_1 + \pi_2 + 2\sqrt{\pi_1(\pi_1+\pi_2)}) \qquad (13)$$

The absolute lower bound in inequality (13) is derived by setting $K = \sqrt{\pi_1(\pi_1+\pi_2)}$.

Notice that there is no need to analyze the trade off between B_2 alone or B_1 and B_2 together. As can be seen in the analysis for the

"exception" segments, that trade off is dominated by the others. The optimal strategy for promoting to the "exception" segments (3 and 6) is summarized in Table 2.

[TABLE 2 ABOUT HERE]

The essence of the strategic implication is that, as relative profits grow, the manufacturer can expose ever more brand share to promotional activity. Since $\pi_2 < \pi_1$, the optimal strategy is to promote π_2 for the very lowest profit ratios. As the profit ratio rises, it becomes, at some point, optimal to expose π_1 worth of preference intensity to promotion. At some still higher value, it is optimal to expose $\pi_1 + \pi_2$ worth of preference intensity. The precise points at which these changes occur are a function of the particular values of π_1, π_2 and K for "exception" segments. For "stockpile and exception" segments, the values of α, β and n also affect the optimal strategy.

[FIGURE 3 ABOUT HERE]

Figure 3 graphically displays the strategy suggested in Table 2. For this example we assume that the number units stockpiled, n, equals 2; the probability that a comptitor is on promotion when manufacturer 1's brands are not being promoted, β, equals (Z-1)a; the number of competitors, Z-1, equals 5; and the probability that manufacturer 1 is promoting, a, equals 0.1. The same (π_1,π_2) pairs are used in Figure 3 as were used in Figure 2.

Comparing Figures 2 and 3, we see that all break points have been shifted to the right. This suggests that, all else being equal, a manufacturer should expose less share to promotion in those markets with a relatively higher concentration of stockpilers.

Sequencing Promotions

We now relax the assumption that K, the expected level of competitive promotional activity, is constant across time. We instead assume the existence of two promotional seasons. Competitive promotional activity is expected to be higher in season I than in season II ($K_I > K_{II}$).

Given our assumption that there exists an upper limit on α, manufacturer 1 probably won't want to promote any brand continuously. The manufacturer's decision, then, is to decide whether to promote B_1 in season II and B_2 in season I or B_1 in season I and B_2 in season II.

To answer this question we must examine the relative profitability of the two possible promotion schedules (B_1 in season II and B_2 in season I versus B_1 in season I and B_2 in season II). For the question of sequencing, it turns out that the pivotal condition is the same for both the "exception segments" (3 and 6) and for the "exception and stockpile" segments (4 and 7). In general, for it to be optimal to promote B_1 in season II and B_2 in season I, the following must hold:

In segments 3 and 6,

$$P^{I}_{1,\sim 2} + P^{II}_{\sim 1,2} < P^{I}_{\sim 1,2} + P^{II}_{1,\sim 2}$$

and, in segments 4 and 7,

$$\hat{P}^{I}_{1,\sim 2} + \hat{P}^{II}_{\sim 1,2} < \hat{P}^{I}_{\sim 1,2} + \hat{P}^{II}_{1,\sim 2}$$

TABLE 2. OPTIMAL PROMOTIONAL STRATEGY TO SEGMENTS 4 AND 7

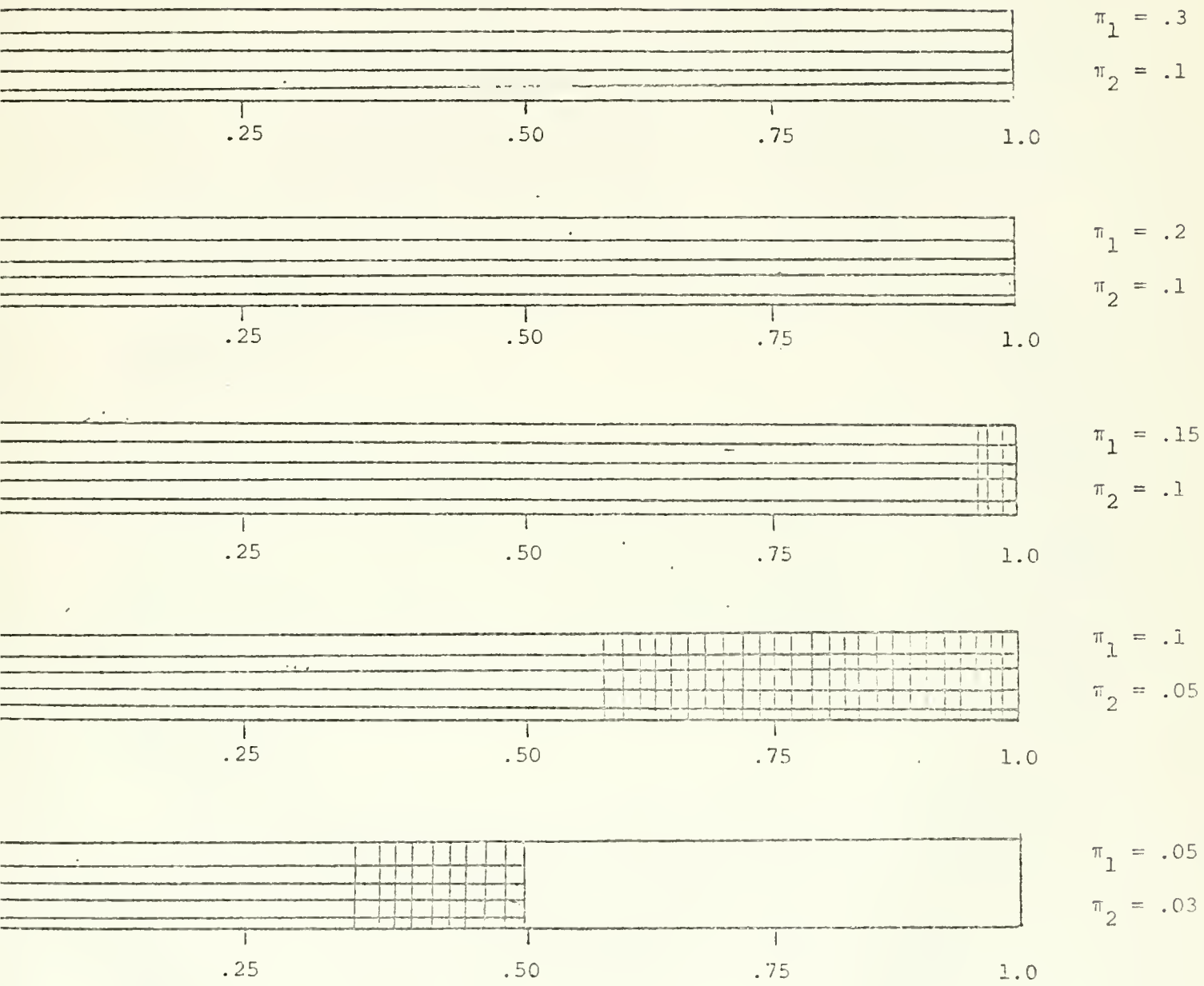
Conditions on M_2/M_1	Brand(s) to Promote
$\frac{M_2}{M_1} < [1 + (1-\alpha-\beta)(\frac{\beta}{\alpha})(\frac{n-1}{n})](\pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K})$	B_2
$[1+(1-\alpha-\beta)(\frac{\beta}{\alpha})(\frac{n-1}{n})](\pi_1 + \pi_2 + K + \frac{\pi_1\pi_2}{K}) < \frac{M_2}{M_1}$	B_1
$< [1 + (1-\alpha-\beta)(\frac{\beta}{\alpha})(\frac{n-1}{n})](2\pi_1 + \pi_2 + K + \frac{\pi_1(\pi_1+\pi_2)}{K})$	
$\frac{M_2}{M_1} > [1 + (1-\alpha-\beta)(\frac{\beta}{\alpha})(\frac{n-1}{n})](2\pi_1 + \pi_2 + K \frac{\pi_1(\pi_1+\pi_2)}{K})$	B_1 and B_2

FIGURE 3:

STRATEGY FOR PROMOTION TO SEGMENTS 4 AND 7 AS A FUNCTION OF M_2/M_1

FOR DIFFERENT VALUES OF π_1 AND π_2

uming n = 2
. . $\beta = (Z-1)\alpha$
$Z = 6$
$\alpha = .1$



Promote only B_2

Promote only B_1

Promote both B_1 and B_2

As can be seen in Theorem 1 in the Appendix, both of the above conditions are equivalent to:

$$\frac{\pi_1}{\pi_1+K_I} + \frac{\pi_2}{\pi_2+K_{II}} < \frac{\pi_1}{\pi_1+K_{II}} + \frac{\pi_2}{\pi_2+K_I} \qquad (14)$$

The essence of this condition is that the sum of the promotional sales peaks resulting from the promotion of B_1 in season II and B_2 is season I must be greater than the sum of the promotional sales peaks resulting from the promotion of B_1 in season I and B_2 in season II. All other perturbations in profits are equivalent under the two schemes.

Theorem 1 in the Appendix demonstrates that condition (14) is equivalent to:

$$K_I K_{II} > \pi_1 \pi_2 \qquad (15)$$

In the rest of this section we will interpret this condition by considering its implications for "parity" brands (those whose shares are roughly equivalent to the expected competitive promotional activity), "small" brands (those whose shares are lower than the expected competitive promotional activity) and "big" brands (those whose shares are greater than the expected competitive promotional activity).

"Parity" Brands

When we say that manufacturer 1 has "parity" brands we mean that $K_I+K_{II} = \pi_1+\pi_2$. This is equivalent to requiring that the average level of promotional activity equal the average share of manufacturer 1's brands. Notice that this is not a statement about the absolute magnitude of $\pi_1+\pi_2$. Rather, the three classes of manufacturers to be discussed (those with "parity," "big" or "small" brands), are classified by the magnitude of the shares of their brands relative to the level of expected promotional competition.

With "parity" brands, the optimal sequencing of promotions depends on the variability of promotional activity across seasons and the discrepancy in preference intensity across manufacturer 1's brands. Specifically, given $\pi_1+\pi_2 = K_I+K_{II}$,

$$K_I-K_{II} > \pi_1-\pi_2 \rightarrow K_1K_2 < \pi_1\pi_2$$

and $$K_I-K_{II} < \pi_1-\pi_2 \rightarrow K_1K_2 > \pi_1\pi_2 .$$

(See Theorem 2 in the Appendix for a proof of this statement.)

Intuitively, this condition suggests that, if expected promotional competition is relatively constant and if π_1 is significantly larger than π_2, B_1 should be promoted against the relatively weaker expected promotional competition of season II. If, however, expected promotional activity varies dramatically across the periods and the brand shares of B_1 and B_2 are similar, then B_1 should be promoted against the relatively stronger expected promotional competition of season I.

"Small" Brands

When the average preference intensity of manufacturer 1's brands is low relative to the average expected promotional competition ($\pi_1+\pi_2 < K_I+K_{II}$), then it is almost always the case that B_1 should be promoted in season II with lower expected competitive promotional activity.

The only exception to this rule comes when the variability of promotional activity is very large. In particular, if

$$K_{II} < \frac{K_I+K_{II}}{2} + \frac{\sqrt{(K_I+K_{II})^2 - (\pi_1+\pi_2)^2}}{2}$$

then manufacturer 1 should promote B_1 in season I which has higher expected competitive promotional activity. (For proof of this see Theorem 3 in the Appendix.)

"Big" Brands

When the average preference intensity of manufacturer 1's brands is high relative to the average expected promotional competition ($\pi_1+\pi_2 > K_I+K_{II}$), then it is almost always the case that B_1 should be promoted in season I.

The only exception to this rule comes when the discrepancy in shares for manufacturer 1's two brands is very large. If

$$\pi_2 < \frac{\pi_1+\pi_2}{2} + \frac{\sqrt{(\pi_1+\pi_2)^2 - (K_I+K_{II})^2}}{2}$$

then manufacturer 1 should promote B_1 in season II.

(FIGURE 4 ABOUT HERE)

In Figure 4 we contrast the sequencing strategies for "big" brands and for "small" brands. The left side of each panel represents the promotional sales resulting from promoting the larger share brand in the most competitive period. The right side of each panel represents the promotional sales resulting from promoting the lower share brand in the most competitive season.

As you can see, for "big" brands, total sales are greater for promoting the larger share brand in the high promotional activity season. This occurs because either the larger or smaller of the "big" brands will capture virtually the entire market in the low promotional activity season. In the high promotional activity season, the large share brand is more effective.

For "small" brands, the larger share brand should be promoted in the low promotional activity season. Neither the larger nor smaller of the "small" brands is very effective in the high promotional activity season. The larger share brand is more effective than the smaller share brand in the low promotional activity season.

SUMMARY, CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this paper we have provided theoretical evidence which is inconsistent with two popular notions.

The first of those notions is that the incentives inherent in promotion expenditures place manufacturers in the prisoner's dilemma. We showed that that notion is true if all brands have approximately equal market shares. Should one brand be much smaller than the others, that small brand's incentive to promote will remain powerful even if all other brands decide to promote.

The second of those disconfirmed notions is that larger share brands benefit more from promotions. We found quite the contrary. A smaller share brand is less likely to be caught in the "prisoner's dilemma." Further, when deciding which brand in a line to promote, the optimal strategy favors smaller share brands. This effect is enhanced if consumers stockpile.

We also developed some rules of thumb for managing promotion expenditures. Assuming that it is profitable to promote at all, we have shown that the decision whether to promote B_1, B_2 or both turns on the manufacturer's relative profit $\frac{M_2}{M_1}$. The lower this ratio, the more likely it is that the lower preference intensity brand should be promoted alone. For a somewhat higher relative margin, it becomes more profitable to promote the higher preference intensity brand. At still higher levels of M_2/M_1, it is optimal to promote both brands.

Assuming that the expected level of competitive promotional activity varies across time and assuming that no one brand should be continually promoted brings up the problem of sequencing promotions. A manufacturer whose

FIGURE 4: THE RELATIVE PROFITABILITY OF DIFFERENT PROMOTION STRATEGIES GIVEN FLUCTUATING COMPETITIVE PROMOTIONAL ACTIVITY

FIGURE 4a: Relative Profitability for "Small Brands."

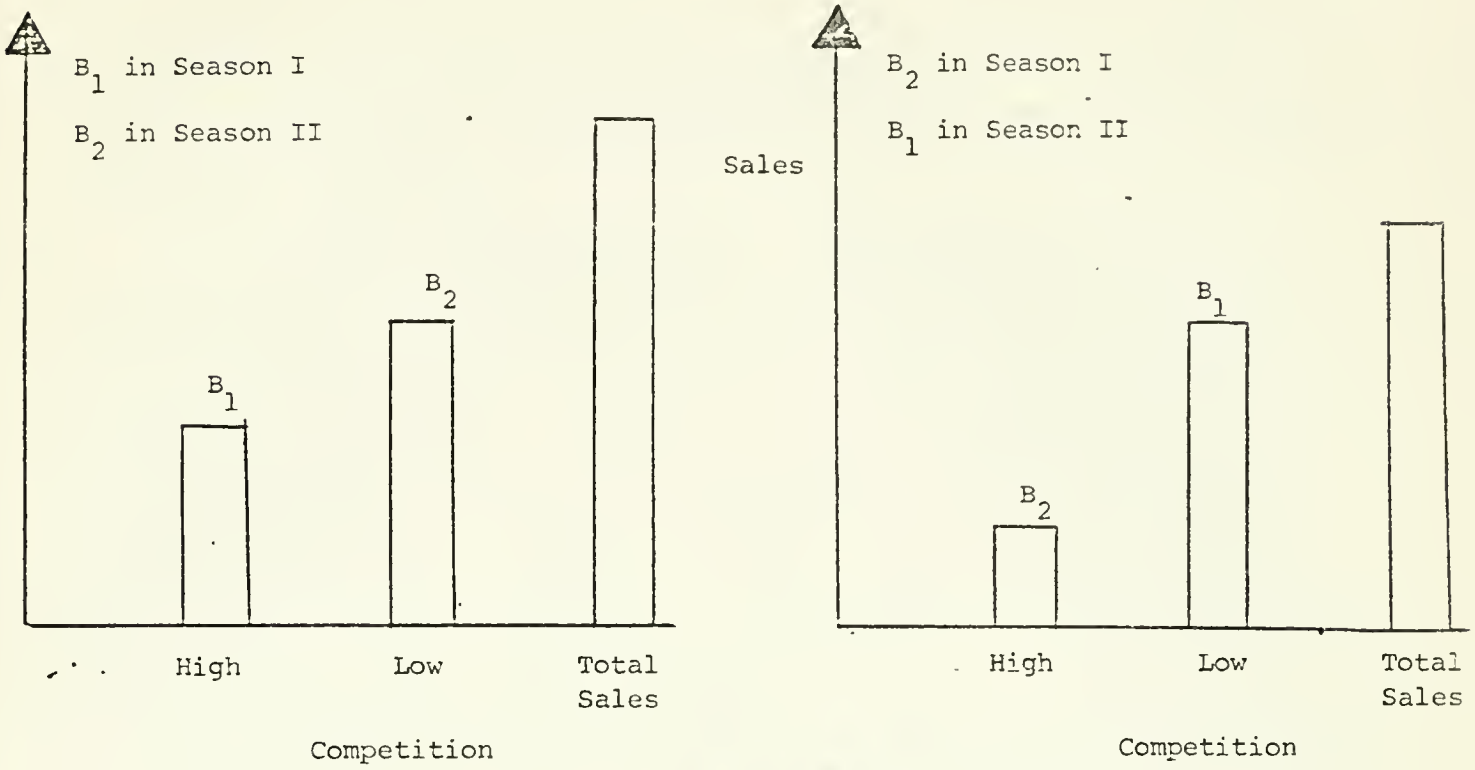
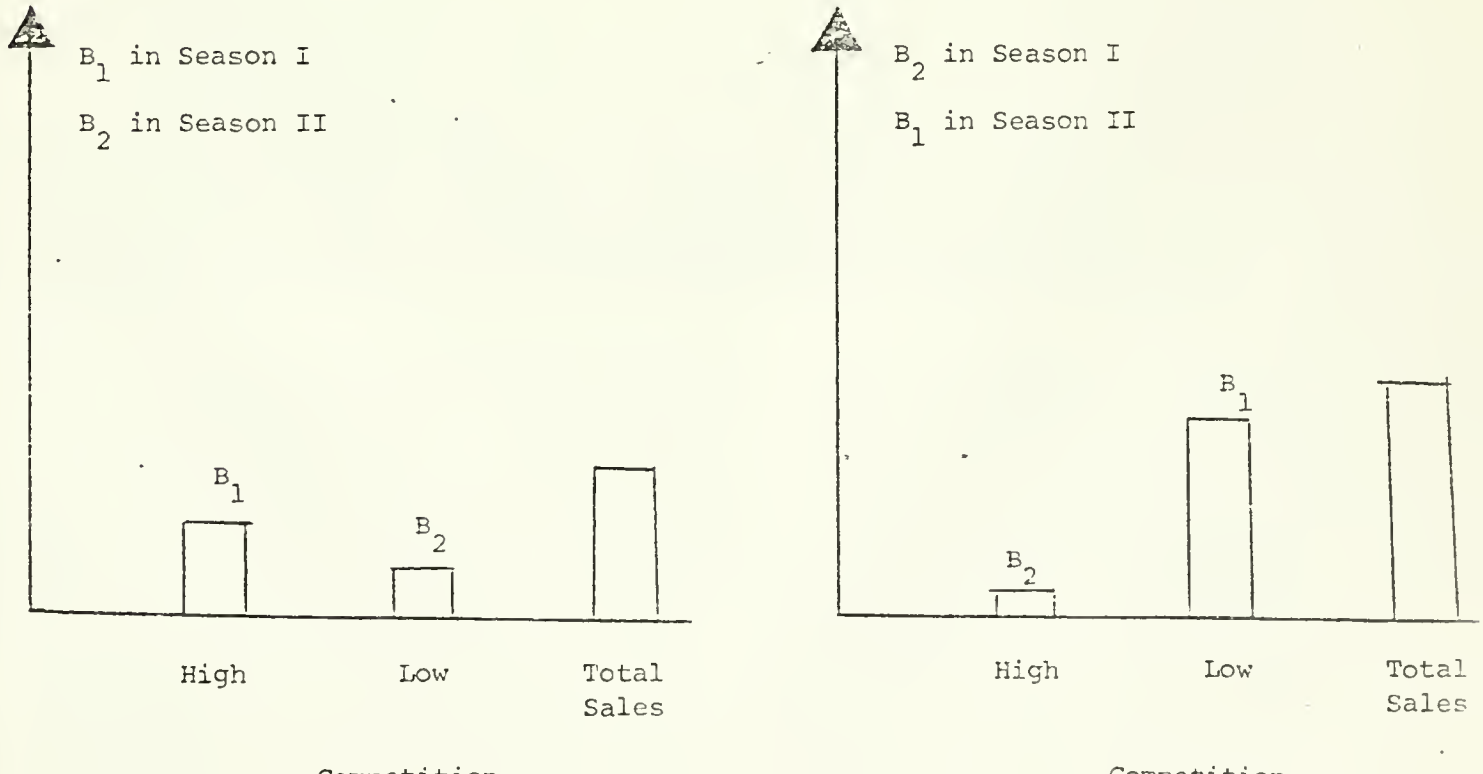


FIGURE 4b: Relative Profitability for "Big Brands."



brands are large relative to competitive promotional activity should promote its larger share brand in the season of more intense promotional competition. A manufacturer whose brands are small relative to competitive promotional activity should promote its larger share brand in the season of less intense promotional competition.

The next step in this research will be to operationalize and test the underlying model. The estimation will be done with UPC scanner panel data. Panel members will be partitioned into eight segments as defined by Figure 1. Examination of non-promoted purchase patterns will distinguish loyal from switching consumers. Comparing an individual's non-promoted and promoted purchase quantities will identify stockpilers. Promoted purchase behavior will expose those loyal consumers who make exceptions and those switchers who restrict their choices to promoted brands.

Having established individual brand preferences, individuals' coefficients of promotional leverage can be estimated and then aggregated. This calibration process will use the first half of the time series data. The second half of the data will then be used to test the model's ability to predict actual market sales.

Having calibrated the model, the manufacturer will be in a position to estimate the profitability of promotion expenditures. Should such expenditures prove profitable, the theory presented in this paper will be applied to develop a promotional calendar for the related product line.

FOOTNOTES

[1]The $60 billion estimate is based on an expected 11% increase in promotion spending beyond the $54.3 billion estimated for 1981. Spending and growth rate estimates are based on Bowman (1981).

[2]"Car Rental Firms Sling it Out with Golf Balls and Tote Bags," Wall Street Journal, December 16, 1982, p. 29.

[3]In Russell D. Bowman, "MC's Second Annual Report on Advertising and Promotion Expenditures," Marketing Communications, August, 1981.

[4]Blattberg, Eppen and Lieberman (1981, p. 117) actually say that "dealing may lead to a zero-sum game." However, their description of the conflicting incentives is consistent with the prisoner's dilemma, not with a zero-sum game.

[5]Here we assume that each of the Z brands would promote alone for a of the time. If, in fact, the promotions overlapped, then the aggregate probability of promotion would be some function of Z and a less than Za. So long as that function increases in a, the spirit of the result as presented in the body of the paper is unchanged.

[6]"Soft Drink Maker Makes Liberal Use of Promotions in Marketing Program," Marketing News, February 18, 1983, p. 10.

[7]So long as the number of consumers in segments 3, 4, 6 and 7 is large relative to the number in segments 1, 2 and 5; this assumption is not a bad approximation of the original model. However, the validity of the proposed rules of thumb comes into question as the relative influence of segments 1, 2 and 5 increases.

[8]This argument follows from the following facts:

$$\frac{\partial \left(\frac{\pi_1}{\pi_1 + K} \right)}{\partial K} = \frac{-\pi_1}{(\pi_1 + K)^2} < 0,$$

$$\frac{\partial \left(\frac{\pi_2}{\pi_2 + K} \right)}{\partial K} = \frac{-\pi_2}{(\pi_2 + K)^2} < 0,$$

and $$\frac{-\pi_1}{(\pi_1 + K)^2} < \frac{-\pi_2}{(\pi_2 + K)^2} \quad \text{for } K < \sqrt{\pi_1 \pi_2}$$

and $$\frac{-\pi_1}{(\pi_1 + K)^2} > \frac{-\pi_2}{(\pi_2 + K)^2} \quad \text{for } K > \sqrt{\pi_1 \pi_2}$$

Furthermore,

$$\frac{\partial^2 \dfrac{K}{(\pi_1 + K)(\pi_2 + K)}}{\partial K^2} = \frac{-2\sqrt{\pi_1 \pi_2}}{[\pi_1 + K(\pi_2 + K)]^3} < 0.$$

[9]First note that

$$\frac{\pi_1}{\pi_1 + K} - \frac{\pi_2}{\pi_2 + K} = \frac{(\pi_1 - \pi_2) K}{(\pi_1 + K)(\pi_2 + K)}$$

$$\frac{\partial \dfrac{K(\pi_1+\pi_2)}{(\pi_1+K)(\pi_2+K)}}{\partial K} = \frac{(\pi_1 \pi_2 - K^2)(\pi_1 \pi_2)}{(\pi_1+K)(\pi_2+K)} = 0$$

$$K = \sqrt{\pi_1 + \pi_2}$$

APPENDIX: Theorems and Proofs

THEOREM 1: When promoting brands B_1 and B_2 with preference intensities π_1 and π_2 ($\pi_1 > \pi_2$) in periods I and II with levels of competitive promotional activity K_I and K_{II} ($K_I > K_{II}$), promotions should be scheduled as follows:

Brand to Promote in Period I	Brand to Promote in Period II		Condition
B_2	B_1	IF	$K_I K_{II} > \pi_1 \pi_2$
B_1	B_2	IF	$K_I K_{II} < \pi_1 \pi_2$

If $K_I K_{II} = \pi_1 \pi_2$, the order of promotion makes no difference.

Proof:

First notice that:

$$P^{I}_{1,\sim 2} + P^{II}_{\sim 1,2} = \left(\frac{\pi_1}{\pi_1 K_I} + \frac{\pi_2}{\pi_2 + K_{II}}\right) \alpha M_2 - (\pi_1 + \pi_2)\alpha M_1$$

$$P^{I}_{\sim 1,2} + P^{II}_{1,\sim 2} = \left(\frac{\pi_1}{\pi_1 + K_{II}} + \frac{\pi_2}{\pi_2 + K_I}\right) \alpha M_2 - (\pi_1 + \pi_2)\alpha M_1$$

Therefore $P^{I}_{1,\sim 2} + P^{II}_{\sim 1,2} < P^{I}_{\sim 1,2} + P^{II}_{1,\sim 2}$ is equivalent to

$$\left(\frac{\pi_1}{\pi_1 + K_I} + \frac{\pi_2}{\pi_2 + K_{II}}\right) < \left(\frac{\pi_1}{\pi_1 + K_{II}} + \frac{\pi_2}{\pi_2 + K_I}\right) \qquad \text{(A1)}$$

We will therefore show that $K_I K_{II} > \pi_1 \pi_2$ implies condition (A1).

Given $K_I > K_{II}$, $\pi_1 > \pi_2$ and $K_I K_{II} > \pi_1 \pi_2$

$$\rightarrow \pi_1(K_I K_{II} - \pi_1 \pi_2) > \pi_2(K_I K_{II} - \pi_1 \pi_2)$$

$$\rightarrow \pi_2(\pi_1 \pi_2) + \pi_1(K_I K_{II}) > \pi_1(\pi_1 \pi_2) + \pi_2(K_I K_{II})$$

$$\rightarrow \pi_1 \pi_2^2 + \pi_1 K_I K_{II} > \pi_2 \pi_1^2 + \pi_2 K_I K_{II}$$

$$\rightarrow \pi_1(\pi_2^2 + (K_I + K_{II})\pi_2 + K_I K_{II}) > \pi_2(\pi_1^2 + (K_I + K_{II})\pi_1 + K_I K_{II})$$

$$\rightarrow \pi_1(\pi_2 + K_{II})(\pi_2 + K_I) > \pi_2(\pi_1 + K_I)(\pi_1 + K_{II})$$

Recall $K_I > K_{II}$, therefore $(\pi_1 + K_{II}) - (\pi_1 + K_I) < 0$

$$\rightarrow \pi_1(\pi_2 + K_{II})(\pi_2 + K_I)[(\pi_1 + K_{II}) - (\pi_1 + K_I)] <$$

$$\pi_2(\pi_1 + K_I)(\pi_1 + K_{II})[(\pi_2 + K_{II}) - (\pi_2 + K_I)]$$

$$\rightarrow \pi_1(\pi_2 + K_{II})(\pi_2 + K_I)(\pi_1 + K_{II}) + \pi_2(\pi_1 + K_I)(\pi_1 + K_{II})(\pi_2 + K_I) <$$

$$\pi_1(\pi_2 + K_{II})(\pi_2 + K_I)(\pi_1 + K_I) + \pi_2(\pi_1 + K_I)(\pi_1 + K_{II})(\pi_2 + K_{II})$$

Divide both sides of the inequality by

$$(\pi_2+K_I)(\pi_1+K_{II})(\pi_2+K_{II})(\pi_1 + K_I)$$

$$\rightarrow \frac{\pi_1}{\pi_1+K_I} + \frac{\pi_2}{\pi_2+K_{II}} < \frac{\pi_1}{\pi_1+K_{II}} + \frac{\pi_2}{\pi_2+K_I}$$

$$\rightarrow P^I_{1,\sim 2} + P^{II}_{\sim 1,2} < P^I_{\sim 1,2} + P^{II}_{1,\sim 2}$$

Hence, if $K_I K_{II} > \pi_1\pi_2$, promote the stronger brand against the weaker promotional competition.

If you assume $K_I K_{II} < \pi_1\pi_2$, all in equalities in the proof are reversed as is the final implication. That is, $K_I K_{II} < \pi_1\pi_2$ implies that the weaker brand should be promoted against the weaker promotional competition.

If $K_I K_{II} = \pi_1\pi_2$, equality of alternatives can be shown by considering:

$$\frac{\pi_1}{\pi_1+K_I} + \frac{\pi_2}{\pi_2+K_{II}} = \frac{2\pi_1\pi_2 + \pi_1 K_{II} + \pi_2 K_I}{\pi_1\pi_2 + K_I K_{II} + \pi_1 K_{II} + \pi_2 K_I} = 1$$

and

$$\frac{\pi_1}{\pi_1+K_{II}} + \frac{\pi_2}{\pi_2+K_I} = \frac{2\pi_1\pi_2 + \pi K_{1\ I} + \pi K_{2\ II}}{\pi_1\pi_2 + K_I K_{II} + \pi_1 K_I + \pi_2 K_{II}} = 1$$

Therefore $P^{I}_{1,\sim 2} + P^{II}_{\sim 1,2} = P^{I}_{\sim 1,2} + P^{II}_{1,\sim 2}$

Q.E.D.

THEOREM 2: Given a_1, a_2, b_1, b_2 and C all between 0 and 1 such that:

$$a_1 > a_2$$

$$b_1 > b_2$$

$$a_1+a_2 = b_1+b_2 = C$$

Then $a_1a_2 > b_1b_2$ iff $a_1-a_2 < b_1-b_2$.

Proof:

Let $a_1 = \frac{C}{2} + c_1$ and $b_1 = \frac{C}{2} + c_2$,

Then $c_1 > 0$ and $c_2 > 0$.

Given that $a_1a_2 > b_1b_2$

$\rightarrow$ $a_1(C-a_1) > b_1(C-b_1)$

$\rightarrow$ $(\frac{C}{2} + c_1)\ (C - \frac{C}{2} - c_1) > (\frac{C}{2} + c_2)\ (C - \frac{C}{2} - c_2)$

$\rightarrow$ $(\frac{C}{2})^2 - c_1^2 > (\frac{C}{2})^2 - c_2^2$

$\rightarrow$ $c_1^2 < c_2^2$

But $c_1 > 0$ and $c_2 > 0$

Therefore $c_1 < c_2$.

$\rightarrow$ $C/2 + c_1 < C/2 + c_2$

$\rightarrow$ $a_1 < b_1$

$\rightarrow$ $2a_1 - C < 2b_1 - C$

$\rightarrow$ $a_1 - (C-a_1) < b_1 - (C-b_1)$

$\rightarrow$ $a_1 - a_2 < b_1 - b_2$.

To show $a_1 - a_2 < b_1 - b_2 \rightarrow a_1a_2 > b_1b_2$, reverse the steps of the above proof.

THEOREM 3: Given a_1, a_2, b_1, b_2, A and B all between 0 and 1 such that:

$$a_1 > a_2$$

$$b_1 > b_2$$

$$a_1 + a_2 = A > b_1 + b_2 = B$$

Then having a_2 such that:

$$a^* = \frac{A}{2} - \sqrt{A^2 - B^2} < a_2 < \frac{A}{2}$$

is sufficient to insure that $a_1 a_2 > b_1 b_2$.

Q.E.D.

Lemma 1: The maximum value for $b_1 b_2 = \left(\frac{B}{2}\right)^2$

Proof:

$$\underset{b_1}{\text{Max}} \; b_1(B-b_1) \rightarrow \frac{\partial b_1(B-b_1)}{\partial b_1} = B - 2b_1 = 0$$

$$\rightarrow b_1 = \frac{B}{2}$$

$$\rightarrow b_2 = \frac{B}{2}$$

Furthermore $\dfrac{\partial^2 b_1(B-b_1)}{\partial b_1^2} = -2$

$\rightarrow$ the inflection point is a maximum.

Lemma 2: If $a_2 = a^* = \dfrac{A}{2} + \dfrac{\sqrt{A^2 - B^2}}{2}$, then

$$a_1 a_2 = \left(\frac{B}{2}\right)^2$$

Proof:

$$a_1 a_2 = A - \frac{A}{2} + \frac{\sqrt{A^2-B^2}}{2} \quad \frac{A}{2} - \frac{\sqrt{A^2-B^2}}{2}$$

$$= \left(\frac{B}{2}\right)^2$$

Lemma 3: a_1a_2 is an increasing function of a_2 for $a_2 \in (0,A/2)$.

Proof:

$$\frac{\partial a_1a_2}{\partial a_2} = \frac{\partial(A-a_2)a_2}{\partial a_2} = A - 2a_2$$

$A - 2a_2 > 0$ for $a_2 \in (0,A/2)$.

Lemma 4: If $B > 0$ then $a^* = \frac{A}{2} - \frac{\sqrt{A^2-B^2}}{2}$ is the interval $(0,A/2)$

Proof:

$$A > B \rightarrow \frac{\sqrt{A^2-B^2}}{2} > 0$$

$\therefore a^* < A/2$

$$B > 0 \rightarrow \frac{\sqrt{A^2-B^2}}{2} < A/2$$

$\therefore a^* > 0$

Hence $0 < a^* < A/2$

Proof of THEOREM 3:

Given $a_1 + a_2 > b_1 + b_2$, we know from Lemma 2 that $a^*(A-a^*) = \left(\frac{B}{2}\right)^2$.

From Lemma 1 we know $\left(\frac{B}{2}\right)^2$ is the maximum value for b_1b_2. From Lemmas 3 and 4 we know that a_1a_2 is an increasing function of a_2 for a_2 in the interval $(a^*,A/2)$.

Hence $a_1a_2 > b_1b_2$ for $a_2 > a^*$.

Q.E.D.

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